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Motivation and background. The calculation of fracture mechanics parameters such as the stress intensity factors (SIFs) for arbitrary three-dimensional structures remains an important task in the structural integrity assessment and damage tolerance analysis. The three-dimensional analyses of crack configurations have received a lot of attention in the last two decades, see e.g. [1, 4]. The finite element method (FEM) and boundary element method (BEM) are widely employed.

The major strength of the FEM is its versatility in handling various classes of engineering problems involving nonlinearity, inhomogeneity, and combining parts modelled under different assumptions. The BEM has strengths in handling problems with stress concentrations. Coupling both approaches is a natural way of combining the strengths of each. In the present context of 3D SIF evaluations within the linear elastic fracture mechanics (LEFM) framework, a sensible use of BEM-FEM coupling consists in modelling using the BEM a region containing the crack, or surrounding the crack front, allowing (i) accurate evaluation of SIFs using quarter-point crack-front boundary elements [4] and (ii) easy modelling of complex crack or crack-front shapes. However, a BEM/FEM coupling based on a conventional collocation BEM formulation leads to nonsymmetric BEM matrices, hence making it impossible to formulate a symmetric and positive-definite BEM-FEM model.

Objective of this communication. Here, an alternative approach for BEM-FEM coupling in 3D LEFM analyses is proposed, aiming at preserving the symmety and positive-definiteness of the overall formulation while retaining the advantages afforded by BEM modelling in the vicinity of the crack front. For this purpose, the Symmetric Galerkin Boundary Element Method (SGBEM) [2], based on a weak formulation (hence in double surface-integral form) of two types of boundary integral equations (respectively displacement-based and traction-based) is used for the BEM-modelled region Ω_c containing the crack front. A linear combination of both types of integral equations (with suitably chosen values of weighting coefficients and trial functions) then allows (upon BEM discretization) to formulate the strain energy W of Ω_c in terms of nodal crack opening displacement DOFs $\{\phi\}$ on the crack Γ and displacement DOFs $\{u\}$ on $\partial\Omega_c \backslash \Gamma$:

$$W = \frac{1}{2} \begin{Bmatrix} \phi \\ u \end{Bmatrix}^{\mathrm{T}} [K_c] \begin{Bmatrix} \phi \\ u \end{Bmatrix} \qquad [K_c] = \begin{bmatrix} K_{\phi\phi} & K_{u\phi} \\ K_{\phi u}^{\mathrm{T}} & K_{uu} \end{bmatrix}$$

where the matrix blocks $K_{\phi\phi}$, $K_{u\phi}$, $K_{\phi u}$, K_{uu} are defined in terms of the bilinear operators arising in the SGBEM and computed by means of double surface integrals over $\partial\Omega_c$. Matrix $[K_c]$ is thus the stiffness matrix of Ω_c expressed in terms of boundary DOFs only. This procedure can be thought of as a condensation on $\partial\Omega_c$ of the stiffness operator for the cracked region Ω_c which completely avoids a domain (FEM) discretisation inside Ω_c. As a result, the cracked region Ω_c can be inserted directly into a FEM-driven analysis and assembled to the global FEM stiffness matrix, i.e. $[K_c]$ is the stiffness matrix of a super-element whose DOFs are all supported on its boundary. The overall analysis is thus driven by the FEM code, with SGBEM computations used solely to set up the super-element stiffness $[K_c]$. It is worth mentioning that a similar approach is independently proposed in [5].

This procedure has been implemented by developing a in-house code (based on the SGBEM implementation of [3]) for the computation of $[K_c]$, which is then read (from a file) and assembled into the commercial FEM code SAMCEF [6]. SIFs distributions along crack front are evaluated from post-processing $\{\phi\}$ on the quarter-point crack front elements. The method has been tested and validated (in both pure SGBEM and coupled BEM-FEM forms) against several problems with known solutions for the SIF distributions, and has yielded accurate results as expected from BEM modelling of cracks. Then, a case study based on real data has been considered, namely a cracked helicopter part (planet carrier of the main gear box), for which the proposed BEM-FEM approach has been applied and the obtained K_I results compared to those produced by a pure FEM approach (Fig. 1).

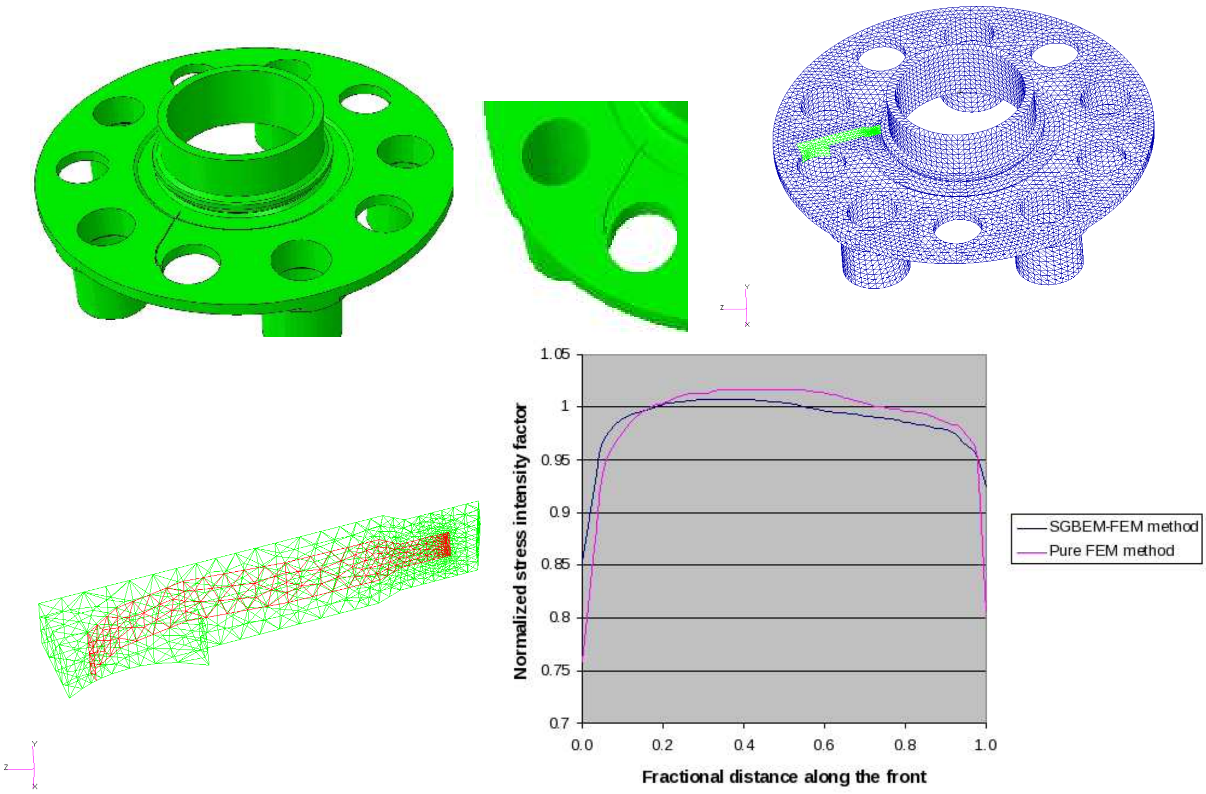

Figure 1: Planet carrier case study: geometry, crack, BEM-FEM mesh (top), BEM mesh with crack, comparison of normalized K_I results obtained by present BEM-FEM coupling and pure FEM (bottom).

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